HADRON STRUCTURE '83

Proceedings of the International Conference on Elementary Particle Physics

November 14—18, 1983
Smolenice
Czechoslovakia

Physics and Applications
Vol. 12

Edited by
I. Lukáč

INSTITUTE OF PHYSICS
EPRC
Slovak Academy of Sciences
Bratislava 1985

ON THE COULOMB + SEPARABLE MODEL OF QUARKONIUM

J. Adam, M. Znojil
Institute of Nuclear Physics, 250 68 Řež, Czechoslovakia

R.F. Bishop
UNIST, Manchester M60 1QD, England

ABSTRACT

Linear confining potential has become a part of the quarkonium physics folklore. In the last decade, a respectable effort has been devoted to its parametrization, modifications, prediction of the $q\,\bar{q}$ decay rates and bound-state spectrum, etc [1]. Within the framework of the Schrödinger equation, the various relativistic corrections were also considered in a perturbative way [2]. Our present intention is to contest this tradition and to suggest an alternative model based on the separable forces.

Let us initiate our consideration in the infinitely heavy quark limit $m_q \to \infty$. Then, the central part $-\frac{4}{3}\frac{\alpha}{r}$ of the non-relativistic potential is acceptable [2] and reflects simply the one-gluon exchange mechanism of interaction. Unfortunately, once we move towards the real masses of the b and c quarks, the static potential picture ceases to be well founded - the relativistic corrections become important and the motion of the gluonic background starts to represent the relevant degrees of freedom [3]. We believe that these corrections (depending on the moments and reflecting the influence of the surrounding medium) have essentially a non-local character.

As a simple model testing such a hypothesis, we shall contemplate here the Schrödinger equation with the separably perturbed Coulombic potential

$$V = -\frac{4}{3}\frac{\alpha}{r} + \sum_{i=1}^{k} |g_i\rangle \beta \langle g_i| + \beta_0 \qquad (1)$$

where the formfactors $\langle r \mid g_i \rangle$ should be short-ranged and sufficiently simple, i.e., say

$$\langle r \mid g \rangle = r^{a+1} e^{-br} \qquad (2)$$

In the simplest case with k = 0, the shifted Coulomb potential (1) cannot agree with the charmonium (bottonium) spectra. This is well known - the simplest proof may be based on the dimensionless observables $a_{n-1} = (E_n - E_{n-1})/(E_{n+1} - E_n)$ obtainable from the quarkonium mass spectrum $M_n(q\bar{q}) = 2m_q + E_n$, n = 1, 2, ... and equal to $a_1^{(c)} = 1.7124$, $a_2^{(c)} = 0.8935$, ... and $a_1^{(b)} = 1.4899$, ... 2 for the charmonium and bottonium $^3S_1$ families and , respectively. The Coulombic values

$$a_{n-1} = \frac{(2n-1)(n+1)^2}{(2n+1)(n-1)^2}, \quad n = 1, 2, \ldots \qquad (3)$$

are listed in Table 1.

TABLE 1. Level spacings for the Coulomb potential

n	1	2	3	4	5	6	7	8
a_n	5.4000	2.8571	2.1605	1.8409	1.6585	1.5407	1.4586	1.3980

Now, the standard addition of the confining force seems to be inevitable. It is not so - let us suggest the following alternative solution of this problem.

Its motivation has been found in nuclear physics where the "ugly" (cored) N-N potential may be replaced by the simple Gaussian, provided that we "forbid" some of its eigenstates. Technically, this was done by Kukulin et al. [4] who proved that the separable force composed of the eigenstates of H ($|g_i\rangle = |\psi_i\rangle$) projects the corresponding energies E_i out of the spectrum (by moving them upwards) in the limit $\beta_i \to \infty$, i = 1, 2, ... k.

We employ the same trick in the present context. Indeed, since

$$a_4 > a_1^{(c,b)} > a_5 , \qquad (4)$$

we obtain a reasonable agreement with experiment (see Table 2 with $|g_i\rangle$ = Coulomb eigenstate (linear combination of (2)), $\beta_i \to \infty$ for i = 1, ..., k, and E_1 and E_2 fitted by α and β_0 in (1).

TABLE 2. Bottonium level spacings predicted by our model

	k=4	k=5	Ref.6	b $\bar{b}$	experimental
$E_2 - E_1$	561	561	585	561	
$E_3 - E_2$	338	364	343	329	
$E_4 - E_3$	219	250	276	224	

Of course, a more precise fit may be obtained with $\beta_5 < \infty$ in (1). By the standard separable potential technique (discussed in fair detail, e.g. in [5]) we may control the energy shift by the choice of the coupling β_5. Numerically, such a procedure is easily performed - a sample of the graphical solution of the related eigenvalue problem is given in Fig. 1.

In the conclusion, let us emphasize that the various sophistications of the present simple model are quite straightforward within the separable-force context. Indeed, the coupled channel calculations as well as an inclusion of the various terms, may be simplified significantly by the projector character of (1). We defer further details to the forthcoming publication.

Acknowledgement. We are obliged to RNDr. A. Adamová for her criticism and inspiring discussions.

References

[1] Quigg C., Rosner J.L., Phys. Rept. 56 (1979) 169;

[2] Choze V.A., Shifman M.A., Usp. Fiz. Nauk 140 (1983) 1;

[3] Voloshin M.V., Nucl. Phys. B154 (1979) 335;

[4] Kukulin V.I., V.G. Neudatchin, Smirnov Yu.F., Nucl. Phys. A245 (1976) 429;

[5] van Haeringen H., Jour. Math. Phys. 24 (1983) 1274;

[6] McClary R., Byers N., prep. UCLA/82/TEP/12.

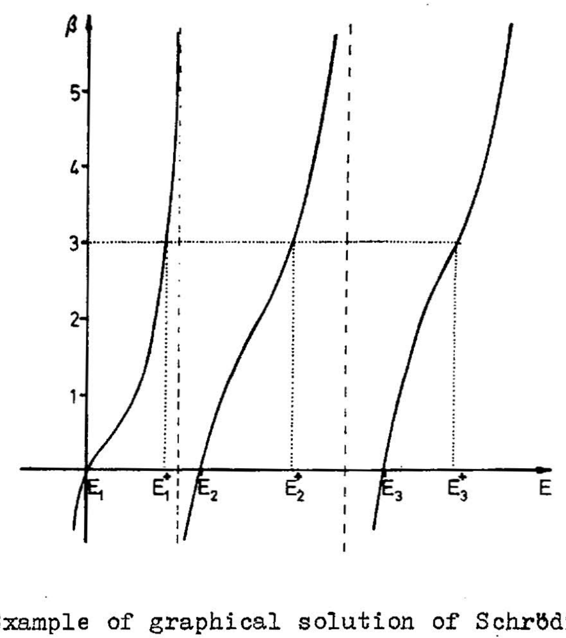

Fig. 1. Example of graphical solution of Schrödinger equation with one term (k = 1) potential of the form (1) and a = 0, b = 2.10^{-5} in (2). E_1, E_2 and E_3 are the (inverted) Coulombic levels and E_1^+, E_2^+ and E_3^+ are the inverse shifted levels for $\beta = 3$. All the quantities are given in units m=h=c=1 .